# Formalising Economics: Social Change, Ideology and Mathematics in Economic Discourse

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In an article by Stigler et al (1995) which studied the use of mathematics in four leading economics journals, it was found articles using neither diagrams nor algebra decreased form 95% in 1892 to an astonishing 5,3% in 1990.<sup>1</sup> In another study by Klamer and Colander (1990) of the five most distinguished doctoral programs in economics in American universities, based on questionnaires given to Ph.D. candidates to answer, and on interviews given by the same candidates, one of the conclusions was stunning. Of those questioned only 3,4 % thought that knowledge about the real economy was very important for success in the doctorate program, while 57% thought that excellence in mathematics was very important. In other words, the students thought that knowledge of techniques and not of the real economy was the basic prerequisite for success in their doctorate programs. How did this state of affairs come about?

Excessive mathematisation and formalisation of economic science has been one of the most important, if not the most important, feature of the development of economic science in the later part of the twentieth century. So pervasive is the penetration of mathematical reasoning into economic discourse that this process was described as the "formalist revolution" by Benjamin Ward in 1972 and this term then adopted by Terence Hutchison (2000) and popularised by Mark Blaug (1999, 2003). Following the recent global economic crisis, including but not confined to the presumed culpability of mathematical modeling of financial markets, increasing number of people, including many leading practitioners of mathematical modeling, have added their voices against this tendency which is considered as driving economics away from real world problems towards proving mathematical theorems. So what were the causes behind this excessive mathematisation of economic science? Why did it happen to such an extent in economics and not in all other social sciences?

<sup>&</sup>lt;sup>1</sup> Similarly Backhouse (1998) in a study of three leading economics journal has found that the number of articles using mathematics (algebra or diagrams or both) rose from zero in 1920 to 40 percent in 1960. If one considers only theoretical articles using algebra then the percentage rises to a staggering 80% during the same period.

is this process reversible? These are some of the questions I try to explore in this paper.

To do so, it is essential to begin with an understanding of the causes and processes that led to this increasingly dominant phenomenon. Scholarship over the last three decades, including Ingrao and Israel (1990), Mirowski (1989, 2002), Weintraub (1985, 2002), Morgan and Rutherford (eds) (1998) has helped in shedding light on some of the factors involved, having set new standards in economic historiography. Although, thanks to these and other works, we are now in a much better position to understand the phenomena of formalisation and mathematisation of economic science, more research is definitely in order to uncover fully what was, and remains, involved. Some accounts tend to rely heavily on one or two factors alone. Most prominent is "the enormous, often uncritical, awe of mathematics in Western Culture" (Lawson, 2003, p. 248); and for (Ingrao and Israel, 1990, p. 34), "The historiography of philosophical thought has long identified the 'mathematisation' of the social sciences as one of the major themes of contemporary culture generated and molded in the rich melting plot of the Enlightenment". So pronounced is this tendency of "awe" that it has led one leading critic to describe it as a form of ideology (Lawson, 2012, pp. 11, 16). Lawson (2003, ch. 10), then, following Ingrao and Israel, identifies the role mathematics in Western Culture, as one of the basic determining factors in the process of the mathematisation of economics. This account although shedding important light to one of the intellectual factors involved, leaves some important questions unanswered. If the importance of mathematics in Western Culture is the basic causal factor, why, for example, did this formalisation process only take place to such an extent after the Second World War? And why has it only come to dominate economics and not other social sciences such as sociology, anthropology and politics (although especially in the latter it has made some important headway)? Lawson (2003, pp. 250-9) attempts to answer the question of why the mathematisation process took off when it did through a natural selection evolutionary process together with a distinctive environmental shift which was favourable to the adoption mathematical methods in economic discourse. This still leaves the question of why, despite this "awe" of mathematics, the latter had to wait for about one and a half century after the publication of Adam Smith's Wealth of Nations before it conquered economics and is still waiting for the full colonisation of other social sciences?

The processes of mathematisation and formalisation of economics are complicated, involving social (including power), economic, intellectual, ideological and institutional factors, and so simple mono-causal explanations are inadequate. To do justice to these multi-causal phenomena, and not to leave the above questions unanswered, all these factors have to be brought to bear by showing explicitly how each of them has had an effect, individually and in combination with the others as well as avoiding a teleology of absence of countervailing factors.

In section 1 we examine the prehistory of the mathematisation process until the 1870s. In it we delineate the role of Newtonianism and liberalism in the formative years of political economy as a separate branch of knowledge by focusing on Smith's attempt to blend the two, and we try to tackle the important question of why all attempts to mathematise economic science utterly failed during this period. This is in fact a question that is left unanswered in the whole literature on the mathematisation of economics which focuses mostly on the evolution of mathematical economics as such. To answer this question, however, one has to look at the broader picture of the evolution of economic science as a whole, something that we attempt to do in this section. In section 2, the first concerted efforts to mathematise economics which took place during and in the aftermath of the marginalist revolution are scrutinised. These involve the works of Jevons and Walras and their followers Edgeworth, Pareto and Fisher through the imitation of the methods of natural sciences (physics and statical mechanics in particular) and prepared the ground of what was to follow about half a century later. These efforts, however, were met with strong opposition both within neoclassical economics, not least through the dominant figure of Alfred Marshall, but also outside neoclassical economics in the works of American institutionalists and the Historical Schools. The inbuilt *ideological biases* of neoclassical theory based on marginalist principles is also exposed. Section 3 examines the changes occurring both within economics through its desocialisation and dehistorisisation and in the natural sciences following the crisis in physics with the appearance of relativity theory and quantum mechanics at the turn of the century and Hilbert's Program in mathematics, which also had an impact on economics.

The 1930s which was probably the most crucial decade in the process of the mathematisation of economics is the subject of section 4. The social, ideological, institutional and intellectual developments that took place during this heated decade,

including the Great Depression and Roosevelt's New Deal, the ideological dominance of socialism over liberalism, the formation of Econometric Society and the Cowles Commission in the 1930s in the U.S.A., and the rediscovery of Walrasian general equilibrium theory, both by economists (John Hicks among them) and, importantly and for the first time, by some top mathematicians in Karl Menger's seminar in Vienna, come under close scrutiny. The ambivalent role of Keyne's General Theory is also examined. The consolidation of this process in the 1940s through the appearance of two milestones, von Neumann's and Morgenstern's Theory of Games and Economic Behaviour (1944) and Paul Samuelson's Foundations of Economic Analysis (1947), is examined in section 5. In the same section the role of the War through its impact in scientific developments and through that in economics is considered. Section 6 tells the story of the 1950s, the decade that has been identified in the literature as the decade during which the formalist revolution took off the ground. Arrow and Debreu were the two most important figures in this process, first through their joint proof of the existence of equilibrium in a Walrasian general equilibrium system in 1954 and, second, through Debreu's book A Theory of Value which appeared in 1959 representing the prime example of mathematical formalism in economics. Section 7 brings to the fore the causal role of ideology in directly shaping developments in economics in the context of the Cold War McCarthyism. Section 8 concludes this paper.

### 1. The Prehistory

The eighteenth century Enlightenment represented the triumph of reason over metaphysics. It "used the growth of scientific knowledge as an antidote against the poison of enforced theological dogma and arbitrary authority in matters of belief" (da Fonseca, 1991, p. 25). The Scientific Revolution, the emergence of (classical) liberalism and the birth of economic discourse were all children of the same cultural environment, the rise of trade and capitalism and the technological advances in Western Europe during the sixteenth and seventeenth centuries, with the former also feeding into the latter in significant ways. Reason (rationalism), individualism, liberalism and universalism were the main Enlightenment values.

The publication of Newton's *Principia Mathematica* in 1687 signified the climax of the Scientific Revolution which took place during the fifteenth and

seventeenth centuries and included the works of the likes of Kepler, Copernicus, Galileo and others. Voltaire was responsible for bringing Newtonianism to France and, by the mid-1750s, the latter was the dominant force in French philosophy, becoming the banner of the French Enlightenment against Cartesianism and Leibnizianism (Schabas and de Marchi, 2003).<sup>2</sup> "Voltaire, in his *Letters on England* (1733), read Newton's achievements as the vindication of Baconian method – of science found on experience, not on mathematical deduction" (Porter, 2003, p. 20). France then became "the scene of Newtonianism's most fruitful developments and greatest triumphs" (Ingrao and Israel, 1990, p. 35). The French Enlightenment has bequeathed upon social sciences, and economic discourse in particular, four major features. <sup>3</sup> First is the idea of the existence of laws governing the social cosmos. Second is rationalism. Third is the concept of harmony and equilibrium. And fourth is the bringing of the individual, emancipated from societal and other fetters of ancient and medieval times, to the fore for the first time in history.

Individualism and individual liberty became the cornerstones of classical liberalism, one of the main philosophical traditions of the Enlightenment, with John Locke and Adam Smith as the two main representatives. "Liberalism was a reaction

<sup>&</sup>lt;sup>2</sup> Descartes and Leibniz were two of the three advocates (the other being Spinoza) of seventeenth century rationalism "in which conclusions are produced by applying reason to the first principles or prior definitions rather than to empirical evidence", and as such was opposed by the empiricist school (http://en.wikipedia.org/wiki/Gottfried\_Wilhelm\_Leibniz). Descartes "propounded the rationalist program of the reduction of all phenomena to matter in geometrical motion", while Leibniz "championed a 'law of continuity', that … nature does not manifest itself in large and abrupt changes. That belief found its expression in the technique of summing sequences of infinite small quantities – that is, in the calculus" (Mirowski, 1989, pp. 16, 18). Newton, along with Descartes and Leibniz, was one of the key figures of the Scientific Revolution. He formulated the laws of motion and universal gravitation which he also applied to celestial bodies, laid the foundations of much of classical mechanics and invented (along with Leibniz) the infinitesimal calculus (http://en.wikipedia.org/wiki/Isaac Newton).

<sup>&</sup>lt;sup>3</sup> "Enlightenment is the process of undertaking to think for oneself, to employ and rely on one's own intellectual capacities in determining what to believe and how to act. Enlightenment philosophers from across the geographical and temporal spectrum tend to have a great deal of confidence in humanity's intellectual powers, both to achieve systematic knowledge of nature and to serve as an authoritative guide in practical life. This confidence is generally paired with suspicion or hostility toward other forms or carriers of authority (such as tradition, superstition, prejudice, myth and miracles), insofar as these are seen to compete with the authority of reason. Enlightenment philosophy tends to stand in tension with established religion, insofar as the release from self-incurred immaturity in this age, daring to think for oneself, awakening one's intellectual powers, generally requires opposing the role of established religion in directing thought and action" (Bristow, 2011).

against mercantilism feudal and aristocratic societies of the *ancient régime*, and stressed the commitment to individual liberty against the coercive powers of the State" (Cockett, 1994, p. 5). Between 1760s and 1820s was the period when the battle of liberalism against mercantilism was fought and won, with liberalism becoming the ruling dogma for the best part of the nineteenth century (until the 1880s) (p. 6)

Ever since the publication of Newton's magnus opus, social scientists have been asking the question: if nature is governed by laws could the same be the same for society? Some have translated this into the following related but different question, "is it possible to apply or adapt the methods of inquiry that have proved so effective in the physicomathematical 'exact sciences' to the study of man's moral, social and economic behaviour?" (Ingrao and Israel, 1990, p. 33). The difference between the two questions, is not semantic and involves very different answers, methods and modes of expression. The latter question leads directly to the "strict analogies" approach between the physical and the social sciences, what in the twentieth century, following Hayek (1942-4), came to be known as scientism. This "strict analogies" approach had a dual manifestation in the social sciences and political economy in particular [was associated with a dual import from the physical to the social sciences]. First, was the import of the mechanical metaphor both in the form of the equilibrium concept and, chiefly, in describing human behaviour - the eighteenth century "manmachine" doctrine associated with La Mettrie's L'Homme Machine (1747),<sup>4</sup> or the late nineteenth century marginalist concept of the "economic man" (da Fonseca, 1991, chs 2,3). Second, and associated with the first, is the use of the principle tool of the physical sciences, that of mathematical reasoning, in the social sciences. Hence, to the extent that the question asked was of the existence or not of social laws, the normal mode of expression employed was the narrative discourse, while the latter question of the appropriate method vis á vis the physical sciences, led invariably to the application of mathematics.

The relationship between economic science and mathematics has been a tormented one. Throughout the so-called classical period and for most of nineteenth century, political economy was a discursive science. Only after 1870, did it start to

<sup>&</sup>lt;sup>4</sup> "Man is a machine, and there is nothing in the entire universe but a single substance diversely mdified" (La Mettrie quoted in da Fonseca, 1991, p. 26).

become a tool-based science through the use of mathematical modeling (Morgan, 2012, ch. 1). One can discern three phases separated by two important ruptures in the process of the mathematisation of economic science (Mirowski, 1991). The first phase can be called the 'prehistory' of mathematisation running from the mid-eighteenth century to around 1870 (Morgan, 2012, p. 6). The second phase, which spans the period between the 1870s and the 1930s, was set off by the marginalist revolution, providing the first important rupture when the foundations of mathematisation of economics were laid. The second rupture which took place between 1930-1960 and is associated with the 'formalist revolution', gave rise to the third phase which was the take-off phase running to the present. During this last period mathematisation became fully consolidated and model-building became the sine qua non of modern economic science (Mirowski, 1991, Blaug, 1999, 2002, Morgan, 2012). The pinnacle of the last two phases was the Walrasian general equilibrium model first in its original form proposed by Walras himself, and then in its more mathematically formal form by Arrow and Debreu (1954) and Debreu (1959).

In their ground-breaking work on the process of mathematisation of economic science, Ingrao and Israel (1990) focus exclusively on the work of mathematical economists, especially those who somehow dealt with general equilibrium models. According to them, "general equilibrium theory originated and developed in the context of a project put forward in varying forms by different scholars to repeat Newton's titanic achievement - i.e. the fulfillment of Galileo's program for a quantitative (mathematical) study of physical processes – in the field of the social sciences" (p. 34). This indeed seems to be the starting point of most, but not all, attempts to apply the mathematical method to economic discourse. However, by opening up the picture to include the development of economic science as a whole, and not of mathematical economics alone, then a very different perspective emerges on developments during the prehistory period of the mathematisation of economic science. It can be broken into two sub-periods, from the mid- to late-eighteenth century, when mathematical reasoning did gain some currency among writers on economic matters, and the period between the end of the eighteenth century and 1870 when mathematical economists failed to make any impact whatsoever. Overall the picture that emerges from this period is one consisting mostly of a systematic *failure* of mathematically-oriented economists to make any substantial inroads into the

dominant economic thinking of the day, i.e. classical political economy. If this is the case, the crucial question to tackle in an attempt to explain the later mathematisation of economics, is why did these earlier attempts fail? This is of crucial importance if one is to avoid teleological arguments based on mathematical awe or otherwise and bring to the fore important factors of resistance to the mathematisation tendency in economics.

To answer this question we need to go back to the beginning of this period. The publication of Newton's Principia Mathematica in 1687 had an impact both on the Scottish and the French Enlightenment. For Scottish moral philosophers, "moral philosophy was to be transformed into an uncompromising empirical science. That, in any case, was David Hume's (1711-1776) message when he presented his Treatise on Human Nature (1739-4) as an 'attempt to introduce the experimental method of reasoning into moral subjects" (Heilbron, 2003, p. 44). Similarly, as seen already, following the importation of Newtonianism in France, one of the basic questions posed by authors of the French Enlightenment was whether social reality is also governed by laws. Among the first to ask this question was Montesquieu who was interested to see whether it was possible to reduce the multiplicity of social phenomena into a few basic underlying laws based on empirical observation, a theme taken over by the leader of the Physiocratic movement Francois Quesnay (1694-1774) for whom society is governed by laws established by the creator. Montesquieu was also among the first to introduce the concepts of harmony and equilibrium into social discourse, which were then also taken over by Quesnay and the English moral philosophers. The first attempts at introducing mathematical reasoning into economics were conducted in France, the most fertile scene of Newtonianism, by members of the Physiocratic movement, be it in the form of Quesnay's Tableau Economique, which amounts to the first major attempt to represent the economy in terms of quantitative flows of the production and distribution of the national product; or Turgot's (1727-1781) use of the metaphor of the circulation of blood and his introduction of the concept of market equilibrium which he borrowed from fluids and mechanics; or, last, through Condorcet's (1743-1794) use of social mathematics and probability calculus

in an attempt to collect and systematise empirical data in order to discern patterns and regularities (Ingrao and Israel, 1990, ch. 2, Heilbron, 2003, pp. 43-7).<sup>5</sup>

During the classical era stretching between 1776, the year of the publication of Adam Smith's Wealth of Nations, and the 1870s, there were several scattered attempts by individual writers to introduce mathematical reasoning into economic discourse.<sup>6</sup> One common characteristic of all these attempts was that they failed to have any impact whatsoever and soon fell into oblivion. So total was this oblivion, that when Jevons and Walras ventured into constructing mathematical models of price determination in the 1870s, they had to (re)invent most of these concepts anew.<sup>7</sup> As Jevons (1957, p. xliii) writes in 1879, "the unfortunate and discouraging aspect of the matter is the complete oblivion into which this part of the literature of Economics has fallen, oblivion so complete that each mathematico-economic writer has been obliged to begin almost de novo". And for Fisher (1925 [1892], p. 109, quoted in Theocharis, 1993, p. viii), "Before Jevons all the many attempts at mathematical treatment fell flat. Every writer suffered complete oblivion until Jevons unearthed their volumes in his bibliography". Similarly for Robbins (1983, p. xi) "the history of mathematical economics before Cournot must in some respects be regarded as consisting of antiquarian curiosa", while Theocharis quotes approvingly Robertson's (1949, p. 535) conclusion that the authors of that period "stand now as more or less isolated figures,

<sup>&</sup>lt;sup>5</sup> "Condorcet stressed the urgency of adapting scientific methods to the analysis of state matters. The moral sciences must 'follow the same method' as the natural sciences; they 'ought to acquire a language as exact and precise, and should reach the same level of servitude'" (Heilbron, 2003, p. 46).

<sup>&</sup>lt;sup>6</sup> The first endeavours to mathematise economics during the classical era include Isnard's (1749-1804) and Canard's (1750-1833) attempts at constructing a general equilibrium model of price determination in the late eighteenth century; von Thünen's (1783-1850) theory of marginal productivity which he applied to factor price determination in 1826; Cournot's (1801-1877) theory of pure price, monopoly and duopoly in 1938; Karl Heinrich Rau's (1792-1870) first introduction of demand and supply curves in 1841; Dupuit's (1804-1860) and Gossen's (1810-1858) development of the concept (but not the word) of marginal utility and attempts to offer a theory of hedonistic calculus in 1844 and 1854 respectively, combined with Dupuit's (1804-1860) introduction of the demand curve (Theocharis, 1983, chs. 5, 7.4, 9; 1993, chs. 4, 6, 7, 9).

<sup>&</sup>lt;sup>7</sup> It is no accident that Walras in the first edition of his *Elements of Pure Economics* fails to acknowledge any of his predecessors other than Cournot, although in later works both he and Jevons (1957, pp. xxviii-xliii) paid tribute to several mathematical economists who wrote before them such as Dupuit, Cournot, Gossen and von Thünen. Walras in fact came into contact with Dupuit's work in 1874, and Jevons 'discovered' Gossen in 1878, only after they had themselves reinvented the concept of marginal utility (Howey, 1973, pp. 25-6)

who cannot be said to have contributed to a current of thought because there is no discernible flow". There seems, therefore, to be unanimous agreement among scholars that the mathematical economists before Jevons and Walras failed to make any impact. The interesting question then is to explain why did this happen and what changed in the 1870s when mathematical economics began to gain some currency among economists?

Following the impact of the publication of Adam Smith's Wealth of Nations in 1776, the search for *mathematical* laws in the economic and social realms considerably subsided. The same, however, did not apply to the quest for *laws* governing the social cosmos. To the contrary, the explicitly stated aim of most classical economists was indeed the search for such laws. Smith wrote at the beginning of the industrial revolution which represented a threshold between the early merchant phase of capitalism of the sixteenth and seventeenth centuries, and the industrial capitalism of the nineteenth century. As the title of his magnus opus suggests, Smith's main aim was to discover "the nature and the causes of the wealth of nations". As a true child of the Enlightenment, Smith's work was a prime instance of the attempt to blend political economy with moral philosophy and the search for societal laws with the values of liberalism. As J.S. Mill (quoted in Riley, 1994, p. xvi) puts in his Principles of Political Economy, "For practical purposes, political economy is inseparably intertwined with many other branches of social philosophy ... Smith never loses sight of this truth" (Mill, POPE, quoted in Riley, 1994, p. xvi). The search for societal laws is associated with the application of the scientific method to the analysis of the social universe, while liberalism is built on the principles of natural liberty and individual freedom. Individualism, economic liberalism and universalism form the main building blocks of his theory. This is reflected in his search for causal factors behind capitalist development, the division of labour and the increase in productivity, in the form of the individual's natural (hence universal) propensity "to track barter and exchange", and his proclivity to pursue his own self-interest, a quest that results in increased social welfare. Having said this, Smith's analysis is full of instances where he deviates from this basic schema, including the wide and multifaceted use of historical analysis and the deployment of more collectivist (class) analysis alongside his individualist arguments.

Similarly Ricardo's aim was to "determine the laws which regulate the distribution ... among the three classes of the community namely, the proprietor of land, the owner of the stock of capital ... and the labourer". For Mill "the political economy informs us of the laws which regulate the production, distribution and consumption of wealth", while Marx's main concern was "to reveal the laws of motion of modern society" (quoted in Milonakis and Fine, 2009, pp. 13, 21). So, although Newtonianism's influence is still present in the form of the quest for social laws, the same does not apply to Newton's, and the physical sciences' more generally, method. Although there are instances where the classicals used some mathematical reasoning, mostly in the form of numerical examples such as in Malthus' law of population, or in Ricardo's demonstration of the laws of distribution in agriculture using farm accounts, or in Marx's demonstration of the transformation of prices into prices of production in Volume III of *Capital*, these are all exceptional cases that help to prove the rule. Classical political economists adopted a discursive (conceptual) mode of expression characterised by "long chains of verbal reasoning", and "argued in terms of principles and laws, not models ... For them, the economy was governed by laws, general and strict, just as the natural world was, and the task of the economist was to discover, or postulate, those laws taking into account of the evidence of the day and of history" (Morgan, 2012, pp. 45-6 and ch.2). Why was this the case, and why did these laws not take the form of mathematical laws?

The nineteenth century was a turbulent period. The industrial revolution of the late eighteenth and early nineteenth centuries alongside industrialisation, technological advancements and the consequent unprecedented increases in productivity, urbanisation and population explosion, also brought about unemployment, poverty, increasing inequalities, and the increasing immisaration of the working people, what came to be known as the "social question". On top of this, industrial capitalism proved to be a very unstable system. The aftermath of the industrial revolution became the scene of recurrent economic crises and social upheavals culminating in the Paris Commune of 1871 and the Great Depression of the nineteenth century between 1873-1879. Although classical liberalism continued its journey not least through the writings of John Stuart Mill, it did not go uncontested (Mill, 1962a, 1962b). The socialist movement which sprung out in the 1820s in the form of the writings of the French socialists (Saint Simon, Fourier and the anarchist

Proudhon) and culminated in the work of Carl Marx, was the child of the adverse consequences of the industrial revolution and the turbulences of nineteenth century industrial capitalism. At the same time the mode of analysis of the main classical thinkers Ricardo and Marx (Mill's was more individualist and more eclectic) was moving away from individualism towards more holistic and collectivist (class) types of analysis influenced, in Marx's case, by Hegelian dialectics, and in the case of the German historical school to more historico-inductive forms of analysis.

In such an intellectual environment, although Newtonianism's influence is still present in the form of the quest for social laws, the same does not apply to Newton's, and the physical sciences' more generally, scientific tools and modes of expression. Classical political economists adopted mostly a discursive (conceptual) mode of expression characterised by "long chains of verbal reasoning", and "argued in terms of principles and laws, not models." (Morgan, 2012, pp. 45-6 and ch.2). Why was this the case, and why did these laws not take the form of mathematical laws?

First, in the late eighteenth century the intellectual atmosphere was changing making it less conducive to the use of mathematical tools outside the natural sciences. This was reflected in the doubts expressed first by the *ideologues* concerning the use of the physico-mathematical method in the social sciences by the Physiocrats, then by pioneers of the emerging new social science, especially Auguste Comte,<sup>8</sup> and by members of the classical school of political economy such as Malthus and Say who were thoroughly against the use of mathematics in social science (Ingrao and Israel, 1990, pp. 54-60). According to Malthus (1986 [1820], p. 1) "the science of political economy bears a nearer resemblance to the science of morals and politics than to that of mathematics". Second, this was the era of what has described as Counter-Enlightenment which he associates mostly with the rise of German Romanticism, which substitutes emotions for Enlightenment's rationalism, and is associated with relativism, anti-rationalism and organicism.

Third, most Enlightenment and classical writers, Quesnay and Smith among them, "still kept their accounts of human behaviour as essentially distinct from the

<sup>&</sup>lt;sup>8</sup> "In the 1820s Comte ... rejected decisively the idea that social science should adopt the same methods as astronomy, physics, or physiology" (Porter and Ross, 2003, p. 4),

explanations of natural processes modeled after the achievements of nineteenth century physics" (da Fonseca, 1991, pp. 33), by refusing to draw a sharp distinction between economic and non-economic motives, considering both to be constitutive of human nature. Smith, in particular, was a prime example of this Enlightenment feature, as manifested, first, in his outright rejection of "Mandeville's attempt to posit an abstract, undifferentiated and all-embracing concept of individual self-love in order to account for both selfish and *prima facie* altruistic forms of human behaviour" (p. 35), and, second, through the identification of sympathy (in his *Theory of Moral Sentiments*) as a pro-social motive of human conduct alongside self-interest.

Fourth, unlike their mathematical counterparts who were mostly trained in the natural sciences, most classical economists, with the exception of Ricardo who was a broker, were either philosophers themselves or had some (initial) training in philosophy: Adam Smith was a moral philosopher, Malthus studied mathematics and natural philosophy, Mill was trained in both political economy and philosophy; and Marx studied law and wrote his doctoral thesis on the philosophy of Epicurus. Fifth, writing either during the course of the industrial revolution or in its immediate aftermath, they were mainly concerned with issues of long-term development and growth. Sixth, and derivative upon the second, they were interested in issues of economic policy and reform (revolution even). Seventh, their focus of attention was the (capitalist) economy which they treated as a dynamic system and which they conceived in its wider social and historical context. Hence social relations and historical processes featured prominently in their analysis.<sup>9</sup> Directly related to this is their relative priority in qualitative over quantitative analysis. Although quantitative questions in the form of Smith's and Ricardo's focus on wages and price of corn or the quantitative aspects of the labour theory of value, are never absent from classical writers, these are at most narrower applications of their qualitative analysis. In effect what most classical political economists sought was the construction of a unified social science in their attempt to explain the workings of the (capitalist) economy. Social relations and historical processes, however, are notoriously difficult to analyse mathematically, as are issues of long-term dynamics and growth in a historical and

<sup>&</sup>lt;sup>9</sup> This is especially true of Smith's and Marx's analysis while Ricardo, whose abstract analysis although social in nature lacked a strong historical dimension is a sort of an exception.

social setting. The same applies to issues of economic policy which require normative analysis (Milonakis and Fine, 2009, ch. 2). Granted all these features, it was natural that classical economists eschewed mathematical reasoning since it was simply unsuitable for the grander purposes at hand. For the same reasons those who strove to mathematise political economy during this period failed utterly in their task.

Similar considerations apply to the fate of the mechanical metaphor in describing human behaviour. The main attempt in this direction was of course Bentham's utilitarian image of human beings as "pleasure-machines" governed by "two sovereign masters, *pain* and *pleasure*". This is perhaps the only instance where the amoral, asocial, Mandevillian individual pure and simple makes its appearance in the whole classical period. This Benthamite, utilitarian, mechanistic creature, did not manage to make much inroads into classical writings, except perhaps in the early writings of J.S. Mill. Even Mill, however, in his mature work distances himself from this reductionist creature of his former mentor, Bentham. According to Mill, "man, that most complex being, is very simple one in his eyes"; granted, he considers "Mr Bentham's writings to have done and to be doing very serious evil" (Mill, quoted in da Fonseca, 1991, pp. 37, 39). As for his utilitarianism, "I regard", he says, "utility as the ultimate appeal on all ethical questions; but it must be utility in the largest sense, grounded on the permanent interest of a man as a progressive being" (Mill, On Liberty, p. 136). This is a far cry from Bentham's selfish, egotistic, "pleasuremachines".

## 2. The First Rupture

Granted this situation during the classical epoch, what changed in the latter part of the nineteenth century to bring about the first self-confident, and relatively successful, attempts to mathematise economic science, first taking shape in the writings of Jevons (1835-1882) and Walras (1834-1910)?<sup>10</sup> First note that classical

<sup>&</sup>lt;sup>10</sup> Menger is not mentioned here, although normally grouped together with Jevons and Walras as the troika of the marginalist revolution, because he did not use mathematics. This, however, makes him a very interesting case from the view point of the theme of this paper since although he used a similar conceptual framework, not least through the deployment of the concept of marginal utility (although

political economy had been in deep crisis from roughly 1850 and under continuous attack from many different quarters and theoretical view points such as the German Historical School, Karl Marx and then the marginalists. Second, one common characteristic of most mathematical economists examined so far is that they were typically trained in some natural science or other.<sup>11</sup> This same attribute is shared by most of the crusaders of new economic thinking who wrote in the marginalist tradition. Of the two pioneers, Jevons studied natural sciences (chiefly chemistry and botany) and moral sciences, and Walras engineering, which, however, he abandoned to devote his time to the study of philosophy, history, literary criticism political economy and social sciences. What is new with them is that first, although they wrote separately, their writings coincided in time, and, second, that they managed to attract followers including the likes of Edgeworth (1847-1926) and Pareto (1848-1923) in Europe, and Fisher (1867-1947) in America. Of the latter Edgeworth was trained in mathematics and statistics, Pareto studied physics and mathematics (although he also had a background in social sciences), and Fisher studied physics and mathematics. Pareto replaced Walras in his Chair at the University of Lausanne, thus forming between them what came to be known as the Lausanne School also, symbolically, known as the Mathematical School, whose central feature was inevitably Walrasian general equilibrium theory.

Third, another novel element is that this is the first time that the transformation of economics into a mathematical *science* on a par with natural sciences becomes a programmatic proclamation. This involves the strict separation of positive analysis from normative analysis, with the latter being preserved for other branches of knowledge such as applied sciences, moral sciences and arts. "The distinguishing characteristic of a science" says Walras (1954, p. 52), "is the complete indifference to consequences, good or bad, with which it carries on the pursuit of pure truth". This contrasts with applied economics which deals with questions of social wealth and individual well-being, and social economics (a moral science or ethics) which deals

the latter does not play the central role it assumes in Jevons and Walras' analysis), he was able to proceed his analysis discursively (see Milonakis and Fine, 2009, ch.13?).

<sup>&</sup>lt;sup>11</sup> Quesnay was a surgeon and physician, Condorcet a professional mathematician, Isnard an engineer and economist, Canard a professor of mathematics, Dupuit studied engineering, and Cournot was a philosopher and mathematician (Ingrao and Israel, 1990, chs 2, 3, Theocharis, 1983, chs 5, 9).

with questions of property, justice and distribution (pp. 60, 76-80, see also Milonakis and Fine, pp. 94-5). So Walras excludes questions of wealth, well-being, property, justice and distribution from the work of an economic *scientist* or, in other words, the sum total of the questions focused upon by classical political economists. The change could not be more dramatic in this respect.

This is also the first time that three main elements of the French Enlightenment, the notion of the individual freed form his societal fetters, the concepts of harmony and equilibrium, and the idea that the economic realm is governed laws, are applied with such force and vigour. But now, this application is associated with either a transformation of the meaning of the concept involved, or with a more specific understanding of it. The conception of the individual and human nature represents an example of the former case, while the precise meaning of the laws governing the social cosmos an example of the latter case. First, then the more rounded conception of the individual and of human nature, nurtured by the representatives of the Enlightenment and classical political economy, including David Hume, Francois Quesnay, Adam Smith and John Stuart Mill, now gives its place to the narrow conception of economic man, or homo economicus. With the advent of marginalism, following in Bentham's utilitarian steps, all non-economic elements in the form of ethical motives found in Enlightenment and classical writers such as altruism (Smith's concept of sympathy in his Theory of Moral Sentiments) disappear from the map of human nature, which is now understood as being moulded by purely selfish economic motives (the self-interest of Smith's Wealth of Nations). According to da Fonseca (1991, p. 47) "The central feature of the metamorphsis of economic agents into 'pleasure-machines' is that they cease being moral persons ...". The cost of all this is "the drastic simplification and homogenization of human agency in economic affairs ... [and] economic action ... is depurated of ethical and psychological elements" (pp. 49, 56). This is in accord with the mechanical metaphor according to which "the theory of the economy proves to be, in fact, the mechanics of utility and self-interest" (Jevons, 1957 [1871], p. 21). This move is essential for the construction of a more abstract type of reasoning lending itself more readily to mathematical analysis by formulating individual action in quantitative-mathematical terms.

At the same time, however, and importantly, "The disengagement of the economic life from morality turns out to be not only a promising starting-point for abstract analysis, it becomes a moral end-point too, that is, a desirable state of affairs ... The 'invisible hand' at work here transmutes *is* to *ought* ... It is a piece of abstraction that easily lends itself to a normative reading" (da Fonseca, 1991, p. 47, see also Milonakis and Fine, chs. 2, 5, Hillinger, 2015, ch. 2.3). Marginalist protestations to the contrary notwithstanding, the moral (ethical, ideological) element crips in *ab initio*, in the very foundations of neoclassical economics. Further, the focus on the (amoral, asocial) individual drives the analysis away from classes and their dangerous, antagonistic connotations.

The methodological individualism deployed by the marginalists and neoclassical economics more generally is a first clear sign of its ideological leanings with liberalism. This is a reflection of the ideological climate of the time. The nineteenth century was the age of liberalism. Following in the footsteps of Adan Smith, J. S. Mill was one of the chief representatives of economic liberalism in the classical era. His liberalism, however, was muted first, much like Smith, by his more rounded individual, "that most complex being", and by his Saint Simonian, socialistic influences, involving "improved ideas of social co-operation and equal justice" (Riley, 1994, p. xvi, Mill, Chapters on Socialism, 1994 [1879]). In political life, economic liberalism as the economic expression of political liberalism reached its peak during the 1870s and 1880s in Britain by becoming the "governing principle of both the Liberal Party, under Gladstone, and the Conservative Party, particularly under Disraeli, up to 1880" (Cockett, 1994, p. 13).

The implicit ideological bias of neoclassical economics does not stop here. On top of the concept of economic man or homo economicus, the other scientific foundations on which neoclassical economics was erected was the concept of equilibrium borrowed, quite appropriately, from static mechanics, the introduction of the change at the margin (marginalist principle) as a basic economic principle of human decision making, and the concept of economic (Pareto) efficiency. Each of these seemingly "neutral" foundation stones of modern economics had inbuilt ideological biases.

To begin with, the concept of equilibrium and perfect competition is far from neutral. Equilibrium implies a harmonious, smoothly running system, free from internally generated interruptions, which if left on its own will always return to a state of equilibrium. This has the essential function, whether intended or not, of driving the analysis away from issues such as economic crises, downturns and depressions, a recurrent phenomenon of nineteenth century economic life at least since the end of the Napoleonic wars. Perfect competition, on the other hand, is a model of the economy close to the liberal ideal laisser-faire capitalism, of free, perfectly functioning markets. Similarly, focusing on decisions taken on the basis of marginal changes in the quantities involved, moves the attention to small, smooth, piecemeal economic and social change and away from long term, revolutionary social change which is the Marxist moto. Last in our list, is the concept of Pareto efficiency which also has important ideological connotations as it is distributionally blind, implying that distribution does not matter. Such an "objective" criterion could help legitimise even the most extreme form of inequality.<sup>12</sup> A similar outcome could be the result of the adoption of the neoclassical (marginalist) theory of distribution first developed by Clark (1892?), according to which each factor of production is rewarded according to its marginal product. The implication is that each factor of production gets its fair share of the product, according to its marginal contribution to production.

At the same time, unlike the classical era, the laws are now expressed in mathematical form. Thus for Walras (1954 [1874], pp. 71-2) "[the] pure theory of economics is a science that resembles the physico-mathematical sciences in every respect". And for Jevons (1957, pp. vii, xxi), "all economics writers must be mathematical so far as they are scientific at all". As Mirowski (1984, 1989, ch. 5) has shown, the first major rupture in the mathematisation of economics is associated with *physics envy* expressed chiefly, but not exclusively, through the adoption of the mechanical metaphor of equilibrium concurrently by different authors. Mirowski

<sup>&</sup>lt;sup>12</sup> Pareto efficient is a point where you cannot make anyone better of without making anyone else worse off. So, for example, in an economy with two agents and two commodities where one agent has all the quantities both commodities available in the economy and the other agent has nothing could be Pareto optimum, provided that any redistribution from agent A to agent B would make agent A worse off!

(1991, p. 147) offers a concise summary on the developments of the era in this respect:

What happened after roughly 1870 was that the analogical barrier to a social mechanics was breached decisively by the influx of a cohort of scientists and engineers trained specifically in physics who conceived their project to be nothing less than becoming the guarantors of the scientific character of political economy: among others this cohort included William Stanley Jevons, Léon Walras, Francis Ysidro Edgeworth, Irving Fisher, Vilfredo Pareto, and a whole host of others. They succeeded where others had failed because they had uniformly become impressed with a *single* mathematical metaphor that they were all familiar with, that of equilibrium in a field of force. They were all so very taken with this metaphor which equated potential energy with "utility" ... that they – sometimes even unaware of each other's activities – copied the physical mathematics literally *term by term* and dubbed the result mathematical economics.

Despite the controversy that this thesis has given rise to, it does help to illuminate one, but only one and possibly not the chief, factor involved in this process.<sup>13</sup>

The self-confidence and self-assertiveness of Jevons' and Walras' statements above are unmistakable<sup>14</sup> - as is their search for scientific credentials in the form of the mathematical method which was to become the *leitmotif* of economic science in the latter part of the twentieth century. But why is this the case? What are the specific features of the "new" economic science that rendered it susceptible to mathematical reasoning? First is that economics is now depicted as a quantitative science. According to Jevons (1957, pp. vii, xxi), since

<sup>&</sup>lt;sup>13</sup> See the articles in De Marchi (ed., 1993) for some critical reactions to Mirowski's *More Heat than Light*. One main line of criticism is that Mirowski focuses on the intellectual factors at the expense of social factors.

<sup>&</sup>lt;sup>14</sup> One can find many other such references in the work of all marginalists, with the exception of Menger. Thus, for Jevons, his exchange equation does "not differ in general character from those which are really treated in many branches of physical science". And for Pareto, "Thanks to the use of mathematics, this entire theory … rests on no more than a fact of experience, that is, on the determination of quantities of goods which constitute combinations between which individuals are indifferent. The theory of economic science thus acquires the rigor of rational mechanics" (both quoted in Mirowski, 1984, pp. 363, 364).

economics "deals with quantities, it must be a mathematical science". This is possible because capitalism is the first economic system where the economy assumes some sort of autonomy from the other spheres of social reality, and where commodity production and market relations become ubiquitous, transforming social relations into quantitative relations between commodities. Second, is the adoption of deduction as the chief method of economic investigation by all the marginalists. Although the application of the deductive method<sup>15</sup> does make the use of mathematics mandatory in any way,<sup>16</sup> it does facilitate the use of mathematics as the two share the same logical structure. According to Debreu (1986, p. 1261),

Deductive reasoning about social phenomena invited the use of mathematics from the first. Among the social sciences, economics was in a privileged position to respond to that invitation, for two of its central concepts, commodity and price, are quantified in a unique manner, as soon as units of measurement are chosen.

Third, the focus of attention now shifts away from issues of development and distribution involving social relations and historical processes taking place in historical time, to the atemporal, static issue of price determination analysed in terms of equilibrium, a concept itself borrowed from static mechanics.<sup>17</sup> Fourth is the shift away from issues of long-term economic and social change and dynamics to (very) short-run individual maximisation and decision making at the margin. But the very notion of a marginal magnitude is a mathematical concept involving differential calculus. Hence mathematical reasoning becomes indispensible to economic

<sup>&</sup>lt;sup>15</sup> Deduction is defined as the method of developing a theory by starting with given assumptions and premises and, through syllogism and the use of the rules of logic, moving to what are effectively conclusions predetermined by the starting points (Milonakis and Fine, 2009, p. ??).

<sup>&</sup>lt;sup>16</sup> David Ricardo and the representatives of the Austrian School (especially Menger and von Mises) are two prime examples of authors using the deductive method while eschewing the use of mathematics.

<sup>&</sup>lt;sup>17</sup> As Pareto (1987, p. 490, quoted in Hodgson, 2012, p. xvi) puts it, "rational mechanics gives us the first approximation to the theory of the equilibrium and of the movements of bodies … Pure economics has no better way of expressing the concrete economic phenomenon than rational mechanics has for representing the concrete mechanical one. It is at this point that there is a place for mathematics. … It therefore appears quite legitimate to appeal also to mathematics for assistance in the solution of the economic problem".

theorising. In order to arrive at such a state of affairs it was essential that economics got rid of all "pre-scientific vestiges" such as the social, the historical and the normative element (Milonakis and Fine, 2009, ch. 6).

There is no question then that the marginalist revolution laid the foundations which made the mathematisation of economic science possible: rationality and individual maximisation, equilibrium and marginal analysis, were all tools used, even if not invented, by the early marginalists. It was not, however, until half a century later that the full potential of the mathematisation process - that started as a sort of mini-movement with marginalism, as opposed to simple efforts by individual authors which had hitherto been the case - was realised. Why was this the case and what are the causes of this delay in the forward march of marginalism and of establishing the mathematical mode of expression as the chief tool of economic reasoning?

To begin with developments in the real economy were moving in the opposite direction to a perfectly functioning market as depicted in the model of perfect competition. This is the era of the rise of large corporations, trade unions and labour law as well as technological dynamism, all of which were outside the purview of neoclassical economics except for Marshall's analysis beyond the organon.

At the same time, starting from the 1880s, there is a discrete change in the ideological climate, what has been described as a change from the "age of individualism" to the "age of collectivism". This era was stamped by the foundation of the Fabian Society in Britain in 1884, "the first organisation to formulate and aggressively and successfully promote a coherent intellectual justification for the extension of the power of the State in pursuit of certain specific aims", which started to be implemented in the early twentieth century through the introduction of a range of welfare measures including old age pensions and social insurance (Cockett, 1994, pp. 14, 15). "This steady march of collectivism was ... given a tremendous fillip by the first World War, when the demands of war saw the final buckling of the Victorian liberal state, giving way to an unprecedented degree of central control and central economic planning, measures which were ... supported and carried through by politicians of all parties ..." (p. 15-16). At the same time, even more radical changes in the same direction were taking place in Russia following the Bolshevik revolution of 1917. It would not be far off the mark to say that the interwar years in particular

were stumped by the ideological triumph of various forms of socialism and collectivism. As one leading liberal commentator writing in the aftermath of the Russian revolution puts it, "Socialism is the watchword and the catchword of our day. The socialist idea dominates the modern spirit. The masses approve of it. It expresses the thoughts and feelings of all; it has set its seal upon time. When history comes to tell our story it will write above the chapter "The Epoch of Socialism" (von Mises, 1981, p. 15). Free market ideology then was on the retreat, leaving little space for the further development and elaboration of neoclassical economics, the advocates of which were also skeptical about the ability of free markets to deliver the goods (Burgin, 2012, p. 15). Faith in free markets was delivered a further blow by the Wall Street crash and the ensuing Great Depression. All in all neither the ideological climate, nor the socio-economic conditions were conducive to the further advancement of neoclassical economics. The latter was simply not in tune with the spirit of the times.

In addition, the first part of the twentieth century was a period of pluralism in economics. Thus in the USA the dominant school of thought was old or American institutionalism with its main representatives being Thorstein Veblen, John Commons and Wesley Mitchell, while in Germany the German Historical School still reigned supreme. What brings those schools together, in addition to their common emphasis on institutions and development, is their common opposition to the marginalist principles and to the use of mathematics in economic discourse (Yonay, 1998, Morgan and Rutherford, 1998, Milonakis and Fine, 2009, chs. 5, 9, 10).

At the same time, the initial reaction both on the part of fellow political economists but also among some leading mathematicians and physicists of the time was anything but enthusiastic. Walras' *Elements of Pure Economics* in particular, which for some was the pinnacle of the marginalist revolution, was initially almost totally ignored by his fellow economists, while the work of early marginalists more generally received a rather cool or even hostile reception by first rate mathematicians and physicists such as Poincaré, Volterra, Bertrand, Levasseur and others for their "abstract schematism and poverty of direct interpretative results" (Ingrao and Israel,

1990, p. 111, also pp. 110-112, 154-173; Mirowski, 1991, pp. 148-9; 1989, pp. 241-250, Lawson, 2003, pp. 269-71 ).<sup>18</sup>

But opposition to the mathematising tendency of marginalist economics also came from within neoclassical economics itself. Another major factor against the forward march of mathematisation was the huge influence of Alfred Marshall's magnus opus *Principles of Economics* (1890) which laid the foundations of neoclassical economics for the next half century and became the chief textbook until its replacement by Samuelson's *Economics* in 1948. For more than half a century Walras' mathematical analysis was buried under the rule of Marshallian economics. Although Marshall was a mathematician, his analysis was mostly verbal and diagrammatic and he eschewed the use of mathematics which he relegated to appendices. Indeed, he was explicitly opposed to the use of mathematics as the chief tool in economic discourse. As he wrote in a letter to Arthur Bowley in 1906 (in Whitaker, 1996, vol. 3, p. 130):

But I know I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypothesis was very unlikely to be good economics: and I went more and more on the rules -(1) use mathematics as a short hand language, rather than as an engine of inquiry. (2) Keep to them till you have done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in four, burn three. This last I did often ... I think you should do all you can to prevent people from using mathematics in cases in which the English language is as short as the mathematical.

Similarly, Marshall was against the use of the mechanical metaphor when it came to human conduct preferring instead the biological analogy and emphasising the human over the mechanical element of individual action and the "pliability of human nature"

<sup>&</sup>lt;sup>18</sup> Poincaré, for example, commenting on Walras' *Elements of Pure Economics* writes that "at the beginning of every mathematical speculation there are hypotheses and that, for this speculation to be fruitful, it is necessary (as in applications to physics for that matter) to account for these hypotheses. If one forgets this condition, then one goes beyond the correct limits". And, at another point, "you regard men as infinitely selfish and farsighted. The first hypothesis may perhaps be admitted in a first approximation, the second may call for some reservations" (both quoted in Lawson, 2003, p. 270). Levasseur was more critical, while Volterra more supportive.

(da Fonseca, 1991, pp. 50-53). None of these features lends itself to mathematical analysis. Granted his own analysis which eschewed the use of mathematics other than as an auxiliary tool, his strong views on the matter, his emphasis on the human over the mechanical element in human conduct, and his wide influence in the course of neoclassical economics, it was natural that neoclassical economics under his influence would be more or less mathematically confined. This is evident in some prominent representatives of neoclassicism during this period such as John Bates Clark, Eugen von Böhm Bawerk, Jacob Viner and Frank Knight who were all non-mathematical (Mirowski, 1991 p. 148). "Consequently, earlier claims to have attained definitive scientific status simply by means of mathematical expression had grown vulnerable and hard to justify. Thus, mathematical discourse occupied a tenuous position within economics in the half-century or so after the rise of neoclassical economics" (p. 149).Be that as it may, Marshall did more than anybody to promote neoclassical economics in his own non-mathematical way. He won a decisive victory over the British Historical School which involved also personal battles in Cambridge against Cunningham, one of the main representatives of the School. This outcome is not without significance for the later forward march of the mathematisation process in economics. Although Marshall himself was not against the use of history in economics,<sup>19</sup> the main corpus of economics he promoted through his partial equilibrium demand and supply analysis and which became the core of neoclassical economics for the next period was both atemporal (static) and ahistorical in character. And the initial marginalisation of the (British) historical economists and their eventual excision from the economics profession, which induced them to becoming the first economic historians proper, further contributed to the exclusion of the historical element from economic discourse, which in turn eased the expanded use of mathematics in economics (Milonakis and Fine, 2009, chs. 6, 7, 8). The fate of historical economics, which paved the way of the neoclassical dominance in the UK to begin with and globally after the Second World War, is described vividly by William Ashley, a leading member and the first president of the newly founded Economic History Society in 1926 (Ashley, 1927, p. 4)

<sup>&</sup>lt;sup>19</sup> To the contrary, the first two chapters of the first edition of his *Principles* were on economic history while his *Industry and Trade* (1919) was full of historical expositions and illustrations.

The theoretical economists are ready to keep us economic historians quiet by giving us a little plot of our own; and we humble historians are so thankful for a little undisputed territory that we are inclined to leave the economists to their own devices.

The plot, however, was not to remain undisturbed for long as the emergence of a less than humble cliometrics a couple of generations later, following the formalist revolution of the 1950s, testifies.

### 3. Preparing the Ground

#### 3.1 Social Change and Ideology

Perhaps the most crucial period for the shaping of modern economic science in the specific direction it took, was the developments during the middle of the twentieth century. This is true of both the intellectual as well as the social and economic developments of this period. The latter, in particular, were of the outmost importance in this trajectory. They included the 1929 Wall Street Crash and the ensuing Great Depressions of the 1930s; Roosevelt's New Deal; the outbreak of the Second World War; and the advent of the Cold War between the USA and the Soviet Union, following World War II. According to one commentator "events and contingencies in the mid-twentieth century would do more to shape the evolution of American economics than any set of ideas alone" (Bernstein, 2001, p. 64). During the Cold War years, the importance of the ideological factor was also powerfully brought to the fore. In this and the next two sections we trace these developments and try to identify the ways in which they shaped the evolution of economic discourse.

Despite this decisive victory over the Historical Schools, the neoclassical economics in its dominant Marshallian form of the 1920s was not in good shape. In the U.S.A. and in Germany, it had not managed to challenge the dominance of the institutionalists and the historicists, respectively; in France it had not made any substantial headway, as there was general distrust for the concept of utility; and in the U.K. (especially in Cambridge) it was under increasing attack from those such as Joan Robinson and Pierro Sraffa (Mirowski, 1991, pp. 151-2, Morgan and Rutherford, 1998, Yonay, 1998).

But the most devastating blow suffered by neoclassical economics came from the developments in the real economy and especially from the 1929 crash and the Great Depression of the 1930s. For the whole period until the 1929 Wall Street crash, the view that was dominant within neoclassical economics was that markets are efficient, and if left alone they would tend to get back to full employment equilibrium.<sup>20</sup> The result of these beliefs was that, after the 1929 crash, the market was left on its own to cope with the consequences of the crisis. The ensuing deepest crisis and depression of the twentieth century shook the credibility of neoclassical theory and the belief in the self-regulating abilities of the market almost beyond repair. Or so it seemed at the time. Similarly the crisis made plain the total inability of the neoclassical theory of the day to address the phenomenon of the systemic behaviour of the economy given its individualistic approach, let alone to predict the crisis. In a sense, and ironically, the free, unfettered working of the free markets, preached by neoclassical economists of the era (Keynes' "classical economics"), gave a heavy blow to the credibility of the neoclassical doctrine through the Great Depression it brought about. On top of this were other developments in the real economy such as the rise of large corporations, trade unions and labour law as well as technological dynamism, all of which were outside the purview of neoclassical economics except for Marshall's analysis beyond the organon.

The Great Depression brought about the rise of fascism in Europe and Nazism in Germany which prepared the ground for the Second World War. At the same time, in the midst of a deep recession and soaring unemployment throughout the developed world, it was natural that the energies of economists should be devoted to the pressing needs of the day and to economic policy rather than high theory, as is reflected in Roosevelt's New Deal, which, on the one hand, had the effect of revitalizing institutionalism, and on the other hand, of increasing the demand for specialists, a process which was further boosted by the advent of the Great War (Bernstein, 2001, pp. 74-5, see section 5).

<sup>&</sup>lt;sup>20</sup> This belief was based on a two fundamental assumptions of neoclassical theory. First that supply creates its own demand, what is called Say's law. Granted this law, the possibility of crisis is logically precluded. And, second, that unemployment is the result of high wages, and as a result a cut in wages will bring the economy back to full employment.

#### 3.2 Intellectual Developments

According to Weintraub (1983, p. 18), at the end of the 1920s beginning of the 1930s "the times were still hostile to mathematical economics". Be that as it may, the 1930s also witnessed some theoretical developments which shook the edifice of economic science in more than one and often contradictory ways.

Chief expression of these developments was Keynes' *General Theory of Employment, Interest and Money* published in 1936. Being an authentic child of the Great Depression, the main purpose of this book was directed to finding ways for reversing the downward trend in the economy and curing unemployment. Although Keynes, much like Marshall, was trained as a mathematician and the *General Theory* was a theoretical treatise, it was written in typical Marshallian fashion using mostly the discursive mode of expression without, however, eschewing mathematical reasoning altogether. <sup>21</sup> Some simple algebra and diagrams did find their way into the *General Theory*. However, writing in the spirit of the times, Keynes warned against the excessive use of mathematics in economic discourse. As he puts it (Keynes, 1973 [1936], p. 298):<sup>22</sup>

Too large a proportion of recent 'mathematical' economics are merely concoctions, as imprecise as the initial assumptions they rest on, which allow he author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols.

Similarly, Keynes was also against the use of econometrics. "His objection to econometrics stemmed from a conviction that econometric tools could not reveal new economic knowledge, as he wrote in his pointed review of Tinbergen's attempt to test business-cycle theories with statistical tools" (Yonay, 1998, p. 191). At the same time, however, "these historical problems in the economy not only turned economists toward intervention but also created the demand for their services to make concrete plans and suggestions for which the new technical tools of simple mathematical

<sup>&</sup>lt;sup>21</sup> Keynes was after all a student of Marshall in Cambridge where he later became a Professor himself.

<sup>&</sup>lt;sup>22</sup> "It is a great fault of pseudo-mathematical methods of formalizing a system of economic analysis ... that they expressly assume strict independence between the factors involved and lose all their cogency and authority if this hypothesis is disallowed" (Keynes, 1973 [1936], p. 297).

models and statistical techniques were well adapted" (Morgan and Rutherford, 1998, p. 12).

At the same time that Keynes was penning the above lines, another process was going on elsewhere in Europe that was going the shape economic science for the decades to come more decisively than Keynes' writings. Some mathematicians in Vienna were rediscovering Walras' general equilibrium theory in the midst of the deepest recession of the twentieth century, reinvigorating in this way the mathematisation of the dismal science project. Interestingly, and despite his own intentions, Keynes found himself playing a part in this process, albeit indirectly, in more ways than one. But first look at what was going on in this other part of the world.

With the excision of the social and the historical element from mainstream (neoclassical) economic discourse, the road was open for the fuller mathematisation of economic science, notwithstanding Marshall's objections and the resistance of the old institutionalist, the historical schools, and, later on, of Keynes himself. Contrary to the spirit of the time, developments taking place in the 1930s played a pivotal role in the reversal of the trend towards the increasing mathematisation of economic science.

Generally, the mathematisation of economics was meant to make economics more "scientific" and more "rigorous". Before we turn to the developments on this front during this crucial decade, it is instructive to put them into the context of the developments in the physical sciences and the changing meaning of "scientific" and "rigorous". Ingrao and Israel (1990, p. 33), Mirowski (1989, 2002) and Weintraub (1998, 2002), in their quest for an explanation of the mathematisation of economics, all place emphasis on the relation between developments in physico-mathematical sciences and developments in economics (and other social sciences). And there were, indeed, some major changes taking place in the physical sciences at the turn of the twentieth century which were to have an impact on economics.

Throughout the nineteenth century during which Newtonian physics and rational mechanics dominated the scene, mathematics and physical sciences were fellow travelers. In the late nineteenth century through the work of the early marginalists, a new economics body began to emerge imitating their image (Mirowski, 1989). At the turn of the century the physical sciences, and Newtonian

physics in particular, entered a period of deep crisis associated with the evolution of Einstein's relativity theory and the appearance of quantum physics. At the same time, the meaning of formalisation and rigour in the physical sciences was also changing. During the later nineteenth century, the physics envy era of economics when physics was still ruled by static mechanics, the meaning of formalisation and rigour was associated with forging a link between theory and experimental data. As long as the meaning of science was attached to the real world, American institutionalism and historicism with their strong empirical leanings had a good chance of staying at the centre of the stage as, indeed, occurred during the first third of the twentieth century. Following the crisis in the physical sciences, however, and the establishment of the "new physics" of relativity theory and quantum mechanics, Hilbert's Program in mathematics, also called the "Formalist Program', made its appearance in 1918. Mathematics is now "conceived as a practice concerned with formulating systems comprising sets of axioms and their deductive consequences, with these systems in effect taking on a life of their own" (Lawson, 2003, p. 171). As Hilbert (1928, quoted in Weintraub, 1998, p. 1844) himself puts it, "anything at all that can be the object of scientific thought becomes dependent on the axiomatic method, and thereby indirectly on mathematics, as soon as it is ripe for the formation of theory. By pushing ahead to ever deeper layers of axioms ... we also win ever deeper insights into the essence of scientific thought itself, and we become ever more conscious of the unity of our knowledge".

Following this, the notion of formalisation and rigour changes and is now associated with axiomatisation, deductivism and logical consistency or "establishing the integrity of formal reasoning chains" (p. 1843). With Hilbert's Program, mathematics in its new axiomatic form starts to break away from natural sciences and assumes the leading role. "In the sign of the axiomatic method, mathematics is summoned to a leading role in science" (p. 1844). This transformation of mathematics and its assumption of the leading role in science is reflected in the newly-founded self-assertiveness of mathematicians who started applying their abstract tools to subjects which they hitherto considered as lying outside their field of application. In this way a form of *mathematics imperialism* was unleashed: anything that claims to be

scientific can be translated into mathematics, including biology and economics.<sup>23</sup> It is no accident then that this process of the axiomatisation of mathematics and its breaking away from physical sciences reaches a climax during the 1930s, a decade during which economics draws the attention of first rate mathematicians for the first time. This coincides with the *second major rupture* and the beginning of the third, final and decisive phase of the mathematisation of economics. This is the era where the *mathematics envy* associated with increasing axiomatisation, formalisation and abstractness, substitutes for the physics envy of the nineteenth century, eventually giving rise to the "formalist revolution" of the 1950s, to which we now turn our attention (Weintraub, 1998).

## 4. The Roaring 1930s

Although the "formalist revolution" took off in the 1950s, the intellectual developments which took place during the 1930s opened the way for this revolution to take effect. So what were these developments and why were they so decisive for the mathematisation and formalisation of economics?

First, was the (re) definition of economics in terms of scarcity and choice. What all neoclassical writers from Walras and Marshall to Samueslon and Debreu (see below), despite their big differences, hold in common, is their focus on the actions of individuals as their basic unit of analysis. This is the famous homo economicus, Veblen's "lightning calculator of pleasures and pains", or, in other words, the individual stripped of all historical and social context. Until Robbins, however, the definition of economics did not reflect this. Economic was generally defined in terms of its subject matter as the science of wealth or "the study of the ordinary business of life" (Marshall). "Given such definitions it was not clear that economics was a field

<sup>&</sup>lt;sup>23</sup> In their article on the role of the Bourbaki school in economics, Weintraub and Mirowski (1994, p. 246) maintain that "this work is … a case study of how one distinctive mode of mathematics could make inroads into a seemingly distant field and subsequently transform that fields self image, as well as the very conception of inquiry. To be more precise, we shall present a narrative of how the Bourbakist school of mathematics rapidly migrated into neoclassical mathematical economics"

that could be studied with high level of mathematical rigour" (Backhouse, 2010, p. 100). This was put right by Robbins in a non-mathematical text! In his definition, economics becomes the science "which studies human behaviour as a relationship between ends and scarce means which have alternative uses" (Robbins, 1932, p. 15). So the focus of attention as far as the definition of economics is concerned shifts away from the preoccupation with the study of the economy or the market, however defined, or the causes of wealth and material welfare, to individual rationality, scarcity and choice. So economics becomes the science of (rational) choice.

Such a definition of economic science has another, rather surprising, implication. It makes the utilitarian principle redundant. According to Robbins (1932, p. 83?) "The hedonistic trimmings of the works of Jevons and his followers were incidental to the main structure of the theory which ... is capable of being set out and defended in absolutely non-hedonistic terms". To be sure, the mechanical metaphor is still alive and well, the only casualty being the requirement of a theory of human nature and individual behaviour, other than the rationality principle with given tastes and preferences. This line of argument both with respect to the mechanical metaphor and the rationality axiom was first put forward by Pareto in his Manual of Political Economy (1971 [1906]) and his Mind and Society (1935 [1916]) respectively, and was carried to its logical conclusion by Arrow's Social Choice and Individual Values (1951) (see section 7). Pareto was also one of the pioneers, along with Fisher and Cassel, of the concept of ordinal as apposed to cardinal utility. According to him, neoclassical theory rests on "the determination of the quantities of goods which constitute combinations between which the individual is indifferent ... the individual van disappear, provided he leaves us this photograph of his tastes ... The theory of economic science thus acquires the rigor rational mechanics" (quoted in da Fonseca, 1991, p. 54, also pp. 53-55, Milonakis and fine, 2009, pp. 219-224). Although Robbins' definition was not widely adopted at the time, it gradually did so and especially following its inclusion in Samuelson's *Economics* in 1948.

This switch of emphasis had two important implications. First, it greatly facilitated the process of the mathematisation of economics since, given the appropriate assumptions regarding human behaviour, rationality and choice (especially in the absence of uncertainty) are amenable to mathematical modeling. "His definition suggested that rigorous mathematical methods could be at the heart of

economics. For economic science was about working out the implications of the need for choice under conditions of scarcity. Making the best use of scarce resources led directly to the notion that economics was about optimization; hence, that the methods of differential calculus could be used" (Backhouse, 2010, p. 101). The second implication is that given that economics is no longer defined in terms of its subject matter, its scope of application increases enormously, a possibility that was later to be realised starting in the 1950s through the work of Gary Becker and others which set off a process of what came to be known as economics imperialism (Milonakis and Fine, 2009, ch. 12, Fine and Milonakis, 2009).

Third, there was a distinctive change of climate as far as the role of mathematics in economics is concerned. "In fact, the 1920s and 1930s witnessed many changes in the antitheoretical and largely antimathematical climate prevailing among professional economists. A decisive push in this direction was later to be supplied by immigration" (Ingrao and Israel, 1990, p. 249).<sup>24</sup> This is reflected in the differences in the constitutions of the two major economics associations, the American Economic Association (AEA) which was founded in 1885 just a few years after the marginalists wrote their treatises, and the Econometric Association founded in 1930. According to the AEA constitution (quoted in Ingrao and Israel, 1990, p. 146):

We believe that political economy as a science is still in an early stage of development. While we appreciate the work to former economists, we look not so much to speculation as to the *historical and statistical study of actual conditions of historical life* for the satisfactory accomplishment of that development (emphasis added).

The historical overtones, vestiges of the German influence upon economics in America at the time, are evident. There is very little trace of the marginalist proclamations in this statement. The mathematical mode of reasoning in economics was still in its infancy and certainly had not yet penetrated American economics to any serious extent.

<sup>&</sup>lt;sup>24</sup> Note, however, that immigration also worked in the opposite non-mathematical direction through the likes of Schumpeter, Hirshman and many others.

Compare this statement with the following, found in the constitution of the Econometric Society (http://www.econometricscositey.org/society.asp#constitution, quoted in Backhouse, 2010, p. 99):<sup>25</sup>

The main object shall be to promote studies that aim at a unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking similar to that which has come to dominate in the natural sciences.<sup>26</sup>

Similarly, the explicitly stated aim of the Cowles Commission, founded in 1932, was "to advance the scientific study and development ... of economic theory in its relation to mathematics and statistics" (Christ, 1952, p. 11).

So, first, the distinctive change of climate in favour of mathematical reasoning in economics was reflected in the foundation of two institutions (the Econometric Society in 1930 and the Cowles Commission in 1932) and one journal (Econometrica, founded in 1933 and published by the Econometric Society), all devoted to the promotion of mathematics and statistics in economic discourse. What hitherto had been the aims of more or less isolated individual writers, now became the programmatic goal of two newly-founded institutions which were destined to play a decisive role in the transformation of economics. This is the first time that Jevons' and Walras' programmatic statements quoted above become reflected in some official document, hence providing the first step towards the institutionalisation of the use of mathematics in economic discourse. Be that as it may, there was still a long way until the mathematisation of economics was to fully materialise.

At the same time the meaning of "rigour" and "scientific economics" was also changing, in accordance with the developments in mathematics and the physical sciences described above. "Scientific rigour meant logical rigour, dictating that the economics be concerned with developing and analysing precisely specified

<sup>&</sup>lt;sup>25</sup> Ragnar Frisch and Irving Fisher were among the founding members of the Econometric Society (Weintraub, 1983, pp. 80-81).

<sup>&</sup>lt;sup>26</sup> As evidenced by this proclamation, econometrics then had a different and wider meaning including mathematical economics as well as statistical techniques for applied research (Yonay, 1998, pp. 187-8, Backhouse, 1998, p. 85).

mathematical models", in opposition to the meaning attached to the terms before, both in the physical sciences in the nineteenth century, and in economics in the U.S.A. and elsewhere until the 1930s when "scientific rigour meant ensuring that scientific theories were firmly rooted in the real world" (Backhouse, 2010, p. 99).

This change of climate coincides with the influx of a number of mathematicians, scientists and engineers into economics.<sup>27</sup> Importantly, it also coincides with the (re)discovery Walras' general equilibrium theory which, as seen already, had been buried for about half a century under Marshall's flourishing partial equilibrium analysis which was synonymous with the neoclassical economics of the time. What is also new is that this (re)discovery was made not only by economists such as Sir John Hicks but also, and importantly, by some top rate mathematicians who started showing some interest in mathematical economics for the first time, another reflection of the changing climate. Although up until then mathematical economics was mostly practiced by people trained in sciences (physicists and engineers), leading mathematicians and physicists either showed no interest or, when they did, it was in order to provide dismissive comments on work of (mathematical) economists.

The venue for this encounter was Karl Menger's mathematics colloquium in Vienna where some of the top mathematicians of the epoch took part (among them Gödel, von Neumann and Wald).<sup>28</sup> It is there that mathematical economists and mathematicians alike presented their work in mathematical economics in front of an audience of mathematicians. Presenters included Schlesinger, 1932, Wald, 1936, 1937, von Neumann, 1938, and Morgenstern, 1937. Two of them (Wald and von Neuman) were mathematicians while Schlesinger and Morgenstern were economists. Interestingly, what most of them (with the exception of Morgenstern who presented a paper on game theory) offered was some form of reformulation of Walras' general equilibrium system from an axiomatic viewpoint. This was done by "scholars with

<sup>&</sup>lt;sup>27</sup> Among them Ragnar Frisch (1895-1973), Harold Hotelling (1895-1973), Jan Tinbergen (1903-1994), Tjalling Koopmans (1910-1985), Maurice Allais (1911-2010), Kenneth Arrow (1921-) and Gerald Debreu (1921-2004) (see Mirowski 1991, p. 152).

<sup>&</sup>lt;sup>28</sup>Karl Menger was a mathematician and the son of the marginalist Carl Menger.

perfect mastery of mathematical formalism" (Ingrao and Israel, 1990, p. 177).<sup>29</sup> Most mathematicians were still dissatisfied with Walras' system for failing to satisfy the standards of consistency and logical clarity they had come to associate with mathematics in the wake of Hilbert's axiomatisation of mathematics. It was during this time that "the foundations were laid for the theory's *axiomatisation*" a process that reached its climax in Debreu's (1959) *Theory of Value* (pp. 176 and 175-9, 188-197, Punzo, 1991, Weintraub, 1983, 2002, chs 3,4).

It was not only mathematicians that rediscovered Walras' general equilibrium system in the 1930s. Economists themselves also started showing interest again including Hotelling, Lange and Hicks. Lange used the model of general equilibrium, if in a non-mathematical way, during the calculation debate in order to show that some sort market socialism is feasible. The chief moment of this rediscovery by economists, however, was Hicks' Value and Capital, published in 1939, which represented a sort of bridge between Walras' Elements and Samuelson's Foundations of Economic Analysis (1947). Based squarely on the tradition of the Lausanne School of Walras and Pareto, Hicks tried to combine the static theory of prices with the dynamic problems of capital and trade cycles. Having failed to fulfill this (difficult) task, Hicks, much like Walras before him, once again attracted criticism (from Morgenstern in particular) on the grounds of lack of rigour and poor axiomatics. Be that as it may, Hick's book, probably because of its eclectic nature, proved to be an important stepping stone for the reinvigoration of Walras' and Pareto's project, and, through that, for the further mathematisation of economic science not least because of the influence it exerted on two other extremely influential figures of the formalist revolution, Paul Samuelson (1915-2009) and Kenneth Arrow. Both Samuelson's Foundations and, especially, Arrow's work in the 1950s, were attempts to fulfill Hicks' or similar tasks, but on more mathematically rigorous foundations (Weintraub, 1983, pp. 19-21, Ingrao and Israel, pp. 177-8, 235-244, 260-9, 272-7).

<sup>&</sup>lt;sup>29</sup> It is interesting in this respect to note that von Neumann was the link between Hilbert's program in mathematics and the formalisation of economics, later to be joined by Debreu through the Bourbaki group (see below). As Weintraub (1998, pp. 1842-3, notes 9 and 10) puts it, "Hilbert and formalism in economics are inextricably linked through von Neumann". Hilbert worked with von Neumann on quantum mechanics in the 1920s and 1930s, "a period in which von Neumann axiomatised two-person zero-sum game theory".

The 1930s also brought about some real developments in the modeling method. A "new practice" of modeling with "small scale [diagrammatic, algebraic, arithmetic] objects depicting aspects of the economy that can be analysed and manipulated in a number of ways" was introduced (Morgan, 2012, p. 13). This decade, as seen already, also witnessed the beginning of a new branch of quantitative economics, econometrics, through the foundation of the Econometric Society and the appearance of Tinbergen's first econometric model. One other example of this new modeling practice was Frisch's (1933) model of business cycles.<sup>30</sup> Tinbergen was the first to use the term "model", which he borrowed from physics, in order to describe the mathematical and statistical objects he and Frisch were using. The widespread use of models in economics though had to wait a little longer. "The label, the idea, and the use of models became the natural way to work for economists only in the period from the 1940s onwards". By that time "modeling had become the accepted mode of reasoning in economics in the sense that it became 'the right way to reason ... what it is to reason rightly" (Morgan, 2012, pp. 12, 14, 10-4). Indeed, the history of the modeling practice in economics follows closely the history of its mathematisation: the period between 1750-1870 represents the prehistory of modeling with isolated examples of models. The period between 1870-1930 witnesses the "first generation of modelers, a very few economists who regularly made and used such research objects", and the period between 1930 and 1960 was the take off era when modeling assumed widespread use in economics for the first time (p. 6) having been picked up within macroeconomics as well. To this periodisation can be added the period since the 1970s when constructing models became synonymous to doing ("scientific") economics.

Interestingly, as already noted, and despite his own intentions, Keynes found himself playing a role in the revitalising process of the mathematisation of economic science. Although he himself used mostly the verbal mode of reasoning, the macroeconomic categories he introduced including (aggregate) consumption and investment, national income, government spending, demand for money etc., gave a great boost to the further quantification of economics in the form of national income

<sup>&</sup>lt;sup>30</sup> Tinbergen and Frisch won the Swedish Central Bank prize in honour of Alfred Nobel in 1969 for this model-based research (Morgan, 2012, p. 10).

accounting and especially to the econometrics project that was just emerging. This process of creating National Income Accounting methods had started as early as 1920 through the establishment by the institutionalist Wesley Mitschell of the National Bureau of Economic Research (NBER). One of its most prominent members was Kuznets, a statistician and quantitative expert, who played an important role in the quantification project. "The development of a modern system of national income accounting provided a crucial ingredient for the elaboration of a professionalized economics. Its most obvious and immediate impact task was to provision academics and policy makers with the kind of empirical detail and precision that made their work all the more authoritative and persuasive" (Bernstein, 2001, p. 78)

At the same time, no sooner was the *General Theory* published than concerted efforts were made to give his theory a more formal treatment. These included James Meade's (1937) attempt to give "A Simplified Model of Mr Keynes' System" which included "an eight-equation algebraic treatment", Samuelson's similar attempt to model the Keynesian relations in 1939, and, above all and famously, Hick's IS-LL (later IS-LM) attempted formal interpretation of Keynes' analysis of investment and money in his classic article "Mr. Keynes and the Classics" also published in 1937 just one year after the publication of Keynes' *General Theory*. Hicks' analysis, which used two diagrams and three equations, "became the organizing theoretical apparatus of the emerging discipline of macroeconomics" (De Vroey and Hoover, 2004, p. 3, Morgan, 2012, p. 12 and ch. 6).

The common elements in all these endeavours was the attempted reconstruction of some aspect of Keynes' theory in terms of formal (mathematical and/or diagrammatical) models. Indeed, if there is any trace of Keynes in modern economics textbooks, this is through Hick's filter in the form of the IS/LM analysis. So, Hicks is of symbolic importance for our narrative not only because he was one of the forerunners of the formalist revolution through his monograph *Value and Capital*, published in 1939, which influenced both Samuelson and Arrow, but also because he was one of the first to have a go at subsuming Keynes' work to mathematical formalism already from its formative years, through his IS-LL formulation. This process was to continue in the 1940s and 1950s through the work of Klein (1947) which was one of the first books which attempted to mathematise Keynes' theory, Patinkin's *Money, Interest and Prices*, published in 1956, and various articles by

Tobin and Modigliani in the late 1950s giving rise to what Samuelson has called the "neoclassical synthesis" or what has variously been called "bastard Keynesianism" by Joan Robinson, or "hydraulic Keynesianism" by Allan Coddington (1983).<sup>31</sup>

## 5. Consolidation: From Vienna to the Cowles Commission

In the 1940s, the scene of the further developments in mathematical economics moves across the Atlantic to the U.S.A. which was to become the new centre of modern (mathematical) economics, a hegemonic position it still enjoys until today. The role of the Cowles Commission in this process cannot be overestimated. According to Weintraub (1983, p. 18), "It is ... not too far off the mark to identify the Cowles Commission with mathematical economic theory in the U.S." This coincides with the mass emigration of scientists from Europe because of the rise of Nazism and the war: von Neumann, Wald, Menger and Lange were among them. The array of people who served in the Commission is impressive and represents the *dramatis personae* of mathematical economics.<sup>33</sup> Two of the most important works in mathematical economics in the 1950s were Cowles Commission monographs: Koopman's *Three Essays on the State of Economic Science* (1957) and Debreu's *Theory of Value* (Ingrao and Israel, 1990, pp. 255-7, Mirowski, 1991, p.152, see also below).

The 1940s witnessed the next major step in the formalisation and mathematisation of economics. This took the form of two monographs that were meant to play a decisive role in the process: von Neumann's and Morgenstern's

<sup>&</sup>lt;sup>31</sup> For Samuelson the neoclassical synthesis represented "the synthesis of Keynesian demandmanagement policy with the use of the price mechanism to allocate resources, but this position rested on a synthesis of the Keynesian macroeconomic income determination theory with classical or neoclassical macroeconomic principles ... The neoclassical synthesis involved a synthesis of the Keynesian theory of unemployment with 'classical' or 'neoclassical' ideas about how the economy operated at full employment" (Backhouse and Boianovski, 2013, p. 41). On the history of the IS-LM model see the articles in de Vroey and Hoover (eds, 2004). On the neoclassical (or neo-Walrasian) synthesis see Weintraub (1979, ch. 4) and Backhouse and Boianovski (2013, ch. 3).

<sup>&</sup>lt;sup>32</sup> The list includes the likes of Lange (1938-43), Wald (1937-9), Menger (1937), Marschak (1943-8), Haavelmo (1943), Koopmans (1944), Klein (1944), Arrow (1947), Simon (1947), Debreu (1950-1955), and Patinkin .

<sup>&</sup>lt;sup>33</sup> Wald, Menger, Arrow and Debreu were mathematicians and Koopmans studied mathematics and theoretical physics before turning to economics.

*Theory of Games and Economic Behaviour* (1944) and Paul Samuelson's *Foundations of Economic Analysis* (1947). Although these were two different types of books which provided the basis for different research programs in economics and at different times,<sup>34</sup> they both contributed to a great extent in their different ways to the further formalisation and mathematisation of economic science. Von Neumann and Morgenstern's volume represents the first major work in which the new type of axiomatised mathematics was entering economic discourse. According to Debreu (1986, p. 1265) this volume "gained full rights for uncompromising rigor in economic theory and prepared the way for its axiomatisation", a process to which he himself contributed to no lesser extent over the next fifteen years or so. What is more, this was done by one first rate mathematician (von Neumann) and one economist (Morgenstern), thus representing the first book-length incident of the newly-founded form of *mathematics imperialism* in economics which had first taken shape in Menger's seminar over the previous decade.

Samuelson's book, which, as already seen, was influenced by Hick's *Value and Capital*,<sup>35</sup> more than any other single work in economics symbolises the new era in economics. Unlike von Neumann and Morgenstern's book which was a reflection of the latest formalising developments in mathematics, representing the new mathematics envy tendency in economics, Samuelson's chief influence came from the developments in physics, thus representing a step back in the direction of the physics envy of the nineteenth century - only that the type of physics he was imitating was the thermodynamics of late nineteenth century, rather than the mechanics of the earlier nineteenth century.<sup>36</sup> On top of setting the standards of rigour, the concept of

<sup>&</sup>lt;sup>34</sup> Although Samuelson's book made an immediate impact, it took several decades (in the 1970s and 1980s) before game theory became a research project to be reckoned with within economic science.

<sup>&</sup>lt;sup>35</sup> As Samuelson (1998, p. 1381-2) notes, never short of compliments to others, and himself, "Among working economists in the 1930s John Hicks and Ragnar Frisch (two very different economists) got the most attention from me". And, he continues, "when I got to know John and Ursula Hicks well, I said to him: 'I have the best of both worlds. I know your work and I know my own, too'" (p. 1382).

<sup>&</sup>lt;sup>36</sup> According to Weintraub (1991, p. 66 quoted in Lodewijks, 2001, p. 323) "the mathematics of dynamic systems, through Birkhoff and Picard, the applied mathematical analysis of systems from Lotka, the thermodynamics of the late nineteenth century through Gibbs via Wilson, and the confused literature of economic dynamics of the 1930s all shaped the way Samuelson constructed his arguments in the Foundations". And, according to Samuelson's (1998, p. 1376) own recollections, "I was vaccinated to understand that economics and physics could share the same formal mathematical

constrained maximisation he introduced became the economist's chief tool for the next several decades. Samuelson became the symbol of the new era also for another reason. His textbook *Economics*, published in 1948, replaced Marshall's and became the standard textbook for the new era.

During the 1940s the process of mathematisation and axiomatisation of economics was given further impetus by the war, which had both immediate and longterm implications, all in the same technical direction. The direct implication of economists in the war was not without consequences:

Economists not only found their technical expertise useful in making decisions about how to deal with economic shortages (rather than oversupply as in the Great Depression) but turned their techniques to any number of wartime questions, using simple mathematical optimizing models, linear programming techniques, and statistical measurement devices. [Economists were brought in to fight the war directly, planning the optimum bombing-raid design and statistically analyzing firing patterns.] Economists found that by using tool-kit economics and the developing neoclassical technical expertise they could answer questions in very different fields (Morgan and Rutherford, 1998, pp. 12-3).

So, it was the need to mobilise resources through the regulation of the economy in the context of military planning that increased the demand for the economists' skills and offered them the chance to apply their technical prowess, mostly in the areas of resource allocation and strategic decision making. The work of Kuznets and Nathan on national income accounting and that of Koopmans on the most efficient transportation routes were of particular importance in promoting quantitative reasoning in economic discourse. The end result was that "economists emerged from the war covered in glory, perhaps launching the 'economic imperialism' in social sciences over the next half century" (p. 13).

theorems (Euler's theorem of homogeneous functions, Weierstrass's theorems on constrained maxima, Jacobi determinant identities underlying Le Chatelier reactions etc.), while still not resting on the same empirical foundations and certainties".

Ironically, then, it was "the collectivized and centralized war economy that gave neoclassical economists the chance to prove themselves ... Not individualism but rather statism provided the special circumstances ... From the end of the Great War to the advent of the cold war, the American economics had come to age" (Bernstein, 2001, pp. 80, 81-2, 88-89).

Such an outcome may not be as paradoxical as it may seem at first sight. Back in the 1930s in the context of the second phase of the socialist calculation debate, Oscar Lange, in his attempt to show the feasibility of market socialism, had argued that Walrasian general equilibrium system which is supposed to be a decentralised model of the market economy, was in fact a *centralised* system. This was due to what Arrow (1953, p. 43) has called a "logical gap" in the theory. In the absence of any real agent to make a decision on price in a perfectly competitive world, the fictitious auctioneer was called upon to play this role. Granted this, according to Lange (1938, pp. 89-90), a centralised system could actually achieve better results partly because the place of a theoretical construct - the fictitious auctioneer - is taken by an institution with actual, real existence - in Lange's model the Central Planning Board (CPB), in the case of the war economy, the appropriate government body (see also Milonakis, 2003, pp. 99-101).

As Mirowski (2002) has shown, the war also had a big impact on economics through the militarisation of scientific research it brought about, leading to the development and use of advanced mathematical tools, what later became known as operations research, but also artificial intelligence, information theory and cybernetics, which were later on applied to economics leading to a new economic methodology. Mirowski focused on figures such as von Neumann, Simon and Koopmans in order to show the role of the military in the evolution of modern mainstream economics.

However, the impact of these developments was not immediate but gradual, leaving "their footprint upon some important postwar developments in economics such as highbrow neoclassical theory, game theory, rational expectations theory, theories of institutions and mechanism design, the nascent program of 'bounded rationality', computational economics, 'artificial economies', 'autonomous agents', and experimental economics" (Mirowski, 2002, p. 9, also Boland, 2006, Rizvi, 2001: 217). As Weintraub (1991, p. 93, quoted in Lodewijks, 2001, p. 324) puts it,

"Economists began to use tools once those tools became widely used by those working in applied mathematics, engineering, and other applied sciences ... Issues of control, guidance and stability became extremely important in war-related research in applied mathematics ... the period from 1946 to the early 1950s saw wider dissemination of the results ... to the mathematical economics community".

### 6. The Take Off

If the 1930s was the decade when the prelude of the formalist revolution was written and the 1940s the decade of its consolidation, the 1950s was the take-off period when the formalist revolution reached its climax. The pinnacle of this process was Arrow and Debreu's (1954) proof, for the first time, of the existence (but not uniqueness or stability) of a general equilibrium, and the re(in)statement of the Walrasian general equilibrium system in a more mathematically formalised and rigorous way in Debreu's *Theory of Value* in 1959.

Some developments in economic methodology during this decade were of crucial importance in giving a further boost to the increasing abstractness and formalisation of economic theory. Here Friedman's highly influential 1953 essay "On the Methodology of Positive Economics" played a key role. According to Friedman, first, the role of economic theory is not to explain economic phenomena but to make correct predictions and, second, that the assumptions economists make should not necessarily be realistic as long as they make good predictions. Not only that, but the less realistic the assumptions the better are the theories. Economic theories in this conception become instruments of prediction, hence the label instrumentalism. If the predictions are correct then one proceeds as if the assumptions were correct. Despite some heavy criticism coming mostly from economic methodologists, this methodological position was to play a major role in subsequent developments in economic thought, simply because it was convenient. In some sense it liberated economists who began to construct less and less realistic models using more and more sophisticated mathematical techniques without any circumspection. The emphasis

began to be laid on the perfection of mathematical techniques and less on the explanation of economic phenomena.<sup>37</sup>

The mathematical proof of the existence of equilibrium in a Walrasian system by Arrow and Debreu (1954) brought an end to a quest that started some eight decades back in the remote 1870s with Walras' work on general equilibrium. However, it did so at a huge cost. The necessary assumptions for this proof were simply extraordinary, including that "there are forward markets for every commodity in all future periods and for all conceivable contingencies and yet no one holds money as a store of value for more than one period" (Blaug, 1998, p. 11). So, the Arrow-Debreu proof evidently had more to do with mathematical logic than with economic reasoning as such.

The simple most important manifestation of this tendency is Gerald Debreu's book *The Theory of Value* (1959). In this work, the line of research in general equilibrium theory which started through the reworking of the Walrasian general equilibrium system in an axiomatic way by Schlesinger, Wald, von Neumann (all in Karl Menger's seminar), Koopmans, McKenzie and Arrow and Debreu, in the wake of Hilbert's Program in mathematics, reached a climax. Debreu's affiliation with Hilbert's Program came through the Bourbaki group. Back in Europe Hilbert's Program in mathematics suffered a blow because of the proof of Gödel's incompleteness theorems in 1931 which "demonstrated the impossibility of setting up a completely consistent mathematical system" hence showing that Hilbert's program is untenable (Dow, 2003, p. 552).<sup>38</sup> Despite this, the search for a more robust and rigorous mathematics based on axiomatics went on unabated until at least the 1970s.

<sup>&</sup>lt;sup>37</sup> Even so, Friedman's methodological proclamations were only practiced selectively by economists, since, as McCloskey (1985, p. 9) has argued, predictions falsified by the data very rarely led to the abandonment of the theories, reflecting what Hausman (1992, p. 152) has called "the striking methodological *schizophrenia* that is characteristic of contemporary economics, whereby methodological doctrine and practice regularly contradict one another".

<sup>&</sup>lt;sup>38</sup> "The First Incompleteness Theorem provides a counterexample to completeness by exhibiting an arithmetic statement which is neither provable nor refutable in Peano arithmetic, though true in the standard model. The Second Incompleteness Theorem shows that the consistency of arithmetic cannot be proved in arithmetic itself. Thus Gödel's theorems demonstrated the infeasibility of the Hilbert program, if it is to be characterized by those particular desiderata, consistency and completeness" (Kennedy, 2011).

One of the best expressions of this continued quest was the formation of the Bourbaki group, the pseudonym of a group of mathematicians in France whose aim was to reconstruct mathematics on an axiomatic basis.<sup>39</sup> Bourbakism soon spread outside France and especially in America during the 1950s. "Bourbaki came to uphold the primacy of the pure over the applied, the rigorous over the intuitive, the essential over the frivolous, the fundamental over what one member of Bourbaki called 'axiomatic trash'. They also came to define the disciplinary isolation of the mathematics department in post-war America" (Weintraub and Mirowski, 1994, p. 248).<sup>40</sup> According to them, the role of mathematics is "to identify 'the fundamental structures' of operation in mathematics" and thus to construct an axiomatic theory as "a consistent set of definitions", or "an empty schema of 'possible realities"" (Ingrao and Israel, 1990, pp. 284, 285).

The liaison between the abstract formalism of the Bourbaki School and economics was Gerald Debreu who was a student of Cartan, a French mathematician and member of the group.<sup>41</sup> Debreu made no secret of his admiration of the work of the group. As he puts it, "the new levels of abstraction and purity to which the work of Bourbaki was raising mathematics had won a respect that was not to be withdrawn" (quoted in Mirowski, 1993, p. 52). Debreu joined the Cowles Commission in 1950. The reorientation of research at the Cowles Commission from empirical to theoretical/mathematical work had already started under the research director Tjalling Koopmans in the 1940s. The focus of the research was the Walrasian general equilibrium but "counting equations and unknowns in the Walrasian system" was no longer satisfactory (Debreu, 1984, p. 268). According to Debreu (1984, p. 267) himself, "One leading motivation for that research was the study of the theory of general economic equilibrium. Its goals were to make the theory rigorous, to generalize it, to simplify it, and to extend it in new directions". Following the arrival of Debreu "Bourbakism quickly became the house doctrine of the Cowles

<sup>&</sup>lt;sup>39</sup> Diedoriné, Cartan, Weil and Mantelbrot were among them.

<sup>&</sup>lt;sup>40</sup> And for Ingrao and Israel (1990, p. 283).] "The characteristic peculiar to 'Bourbakism' was that of pushing the Hilbertian axiomatic approach to its extreme consequences".

<sup>&</sup>lt;sup>41</sup> Debreu was also influenced by Maurice Allais, the French mathematical economist and Nobel laureate who wrote in the tradition of the Lausanne School of Walras and Pareto (Debreu, 1984, p. 268).

Commission. We would identify the primary philosophical asserting this turning point as Koopman's Three Essays on the State of Economic Science (1957) and Debreu's Theory of Value (1959)" (Weintraub and Mirowski, 1994, p. 263). As Debreu (1959, p. x) puts it in the Preface of his book, "The theory of value is treated here with the standards of rigor of the contemporary school of mathematics", according to which "an atomized theory has a mathematical form that is completely separated from its economic content" (Debreu, 1986, p. 1265). "It seems clear that Debreu intended his Theory of Value to serve as the direct analogue of Bourbaki's Theory of Sets, right down to the title ... Just as with Bourbaki, the problem was to justify the initial identification of the structures" (Weintraub and Mirowski, 1994, p. 265). This took the form of Walras' general equilibrium theory which "in Debreu's interpretation ... loses its status as a 'model' to become a self sufficient formal structure" (Ingrao and Israel, 1990, p. 286). The formalist revolution had reached its peak, as had the total detachment of theory from any claims to realism and real world relevance. "The objective was no longer to represent the economy, whatever that might mean, but rather to codify that elusive entity, the Walrasian system" (Weintraub, 2002, p.121). The cost of theoretical "rigour" and mathematical elegance was indeed immense.

The 1950s represented the pick of a process that had its roots in the marginalist revolution but was reinvigorated since the 1930s giving rise to the formalist revolution. This process involved, first, the transformation of Marshallian economics into mathematical form. This took place through the Robinsonian redefinition of economics as the science of rational choice, hence making mathematical reasoning (in the form of calculus through the maximization and minimization subject to constraints) a *defining* feature of economic science. Second, the rediscovery and the rigorous, mathematically formalised reformulation of the Walrasian general equilibrium theory. Third, the emergence and subsequent development of econometrics (in today's meaning of the term) as a separate branch of mathematical-statistical inquiry. Last, is the subsumption of Keynes' economics to the formalist revolution giving rise to the Keynesian economics of the neoclassical synthesis, also known as bastard of hydraulic keynesianism. The new econonomics thus established represented a sharp break with old-fashioned, Marshallian type neoclassical economics of the interwar period.

In 1958 the Association of Evolutionary Economics was established in the U.S.A. signifying the defeat of institutional economics by mathematical neoclassical economics by granting them "a little plot of their own", in the same way that the establishment of the Economic History Society in 1926 signified the defeat of the Historical Economics by neoclassical economics (see above). One by one the old adversaries of neoclassical economics were being pushed on one side to give room for the new ascending orthodoxy. The formalist revolution was associated with the increasingly dominant position of mathematical neoclassical economics at the expense of pluralism (Yonay, 1998, ch. 9, Morgan and Rutherford, 1998).

#### 7. No ideology please we are economists?

Neoclassical economics and liberalism have been fellow travelers since the inception for the former during the marginalist revolution. This intimate relationship was, to begin with implicit in the inbuilt ideological biases of neoclassical theory favouring free markets. What was implicit in the pre-war neoclassical economics was made explicit during the Cold War era through the attempts to build the explicit theoretical and philosophical foundations of political liberalism and western type democracy. This mostly took the form of rational choice theory. With rational choice theory the parallel journey between rationalism and liberalism, a journey that started during the Enlightenment reaches a climax (Amadae, 2003).

The political and ideological climate during the interwar period and socioeconomic developments such as the birth of the Soviet Union and the Great Depression in the West meant the climate was not conducive for a theory favouring free markets to dominate the scene. Even neoclassical proponents were skeptical of the power of free markets to deliver full employment and prosperity. This continued unabated in post-War Europe. In Britain, during this period, "collectivism, premised on Fabianism and Keynesianism, was the ruling orthodoxy of all parties and governments" (Cockett, 1994, p. 6). The so-called Golden Age capitalism was associated with Keynesian anti-cyclical intervention in the economy and the emergence of the modern welfare state. All this changed dramatically after the Second World War especially in America. Having defeated Nazism and Fascism a new opponent was found in the face of their war allies the Soviet Union. Although this war had military aspects to it, it was a war fought mostly at the ideological level. "In light of the Cold War ideological struggle against the Soviets, this enterprise of securing the philosophical basis of free world institutions was critical" (Amadae, 2003, p. 12). The ideological climate after the Second World War which was highly influenced, if not determined by, the Cold War and McCarthyism, played a pivotal role in this turn of events in economic science. This was done both directly through personal purges and the suppression of certain ideas, and indirectly through the direction of state related funding to specific kinds of research at the expense of others.

The name of the game for US administrators was to counter communism and the collectivist ideology. "Certain kinds of economic analysis were regarded as dangerous, wrongheaded even treasonous ... The impact of McCarthyism was profound and widespread. It played no small part in the enfeeblement of other intellectual traditions" (Bernstein, 2001, pp. 105-6). The most obvious target was of course Marxism because of its direct affinities with the communist ideology. Marxists and left wing economists more generally put their careers at grave risk: "they could be denied promotion, left without search funding, and sequestered from journal editorial boards, all on the basis of a professional 'vetting' regarded as apolitical and rational" (p. ???). But Marxists were not alone in this. Keynes's legacy, which was gaining more and more support even in the USA, was another target of McCarthyist attacks. The problem was of course that Keynes favoured government intervention and a strong state. The result was that anything to do with Keynesianism was associated with the collectivist ideology and a witch-hunt started which lasted throughout the 1940s and 1950s (Goodwin, 1998, pp. 56-62).

A good example of the results of these purges on economic science is given by what may be called the Tarshis-Samuelson incident. Lorie Tarshis published his textbook in 1947 and rapidly became adopted throughout the United States. But them came the reaction: "It was a nasty performance, an organized campaign in which they sent newsletters to all the trustees of all universities that had adopted the book" (Tarshis in Colander and Landreth, 1966, p. 68). Orders started being cancelled, as universities became concerned about, even suffering threats of, loss of endowments.

The result was that "it really died in 1948 or 1949. And then Samuelson's book came out a year later, in 1948". As explained in Milonakis and Fine (2009, pp. 287-293) the main reason for this is that Tarshis took an explicit Keynesian stand: "the importance of avoiding unemployment cannot be overstated ... The upshot of the analysis ... is that unemployment can be cured". He even presented this position as enjoying wide acceptance from left to right. For the McCarthyist witch-hunts, you could not be more Marxist than that. This is only one incident among many during this turbulent period.<sup>42</sup> What was the result? Although Keynesianism did prevail in the United States it was different from Keynes' economics.

Recent scholarship has also brought to the fore the ways in which governments and in particular defense related agencies have exerted a direct influence in the construction of specific theories. Following the war, the military establishment in the US continued to have a direct role in the development of science and of economics in particular influencing decisively the way in which the latter evolved in the immediate post war period. This was done through the direction of funding to specific research programs. Thus most of the funding of the Office of Naval Research, for example, went to mathematical economists, Arrow and Debreu in particular, because research in areas of mathematics and mathematical economics was considered more valuable to the objectives of national defense and security. "The Arrow- Debreu project was noted for its modelling of conflict and cooperation ... whether it be for combat or procurement contracts or exchange of information among dispersed decision makers" (Bernstein, 2001, 97).] In 1945 the Rand Corporation was founded by the US Air Force whose chief aim was to continue the scientific developments of the war, but also to counter the nuclear and ideological threat of Communism. Rand made extensive contributions to operations research, systems analysis, game theory, linear and dynamic programming etc. Its research associates included the likes of Martin Shubik and Oskar Morgenstern who have made extensive contributions to game theory (Goodwin, 1998, p. 64, Bernstein, 2001, pp. 97-100, Anadae, 2003, ch. 1).

<sup>&</sup>lt;sup>42</sup> For more such incidents see Goodwin (1998, pp. 54-62).

What is of direct interest for our purposes is the fact that Arrow's Social Choice and Individual Values, one of the cornerstones of the ongoing formalist revolution and of rational choice theory which "arguably had an ideological use, since it provided an intellectual framework for opposing communism", arose out of his involvement with the RAND Corporation (Backhouse, 2010, p. 145). Specifically, "Arrow was charged at RAND with determining a mathematical expression of Soviet Union's collective utility function that could be useful for game-theoretic strategy computationsof nuclear brinkmanship" (Amadae, 2003, p. 85). In this direction, his main question was whether "it is possible to derive collectively rational group decisions from individual preferences?", to which his answer was that "collectively rational group decisions are logically impossible" (p. 83). This is Arrow's famous "impossibility theorem", which became the cornerstone of the defense of capitalism democracy against Marxism and communism. Granted all this, it is obvious that Arrow, in more ways than one is the child of Cold War, "a high level participant of the Cold War establishment" (p. 85). But Arrow's success was even greater than that. Because he showed the supremacy of western democracy over its rivals using the "objective" tool of mathematics and rational choice theory hence becoming "one of the key creator of the intellectual tradition that would give shape to orthodox American economics during the Cold War period" (p. 85). According to this writer, writing from Arrow's perspective of liberalism in his otherwise excellent book, "The brilliance of Social Choice and Individual Values in part results from the manner in which it molds each of these issues into a coherent theoretical foundation grounding 'capitalist democracy' without raising suspicions of ideological bias, an authoritarian or socialist impulse, or assumptions of the cultural relations of economic laws" (p. 102)! In other words, Arrow then was an high level official of the RAND corporation an organization of the US military establishment, he was charged with the task of opposing communism using the tools of rational choice theory, his theory became the philosophical cornerstone for the of western democracy and yet his theory does not raise suspicion of ideological bias! The reason behind this amazing conclusion lies of course in the objectivity of the mathematical tool even if used for explicitly and directly ideological purposes! This quotation reveals exactly the opposite of what the author intends: the most explicit use of mathematics and rational choice theory as an ideological veil of the ideological role of mainstream economic theory. Amadae actually says so much when he writes that "in its guise as 'objective' or 'value free' social science it is

difficult to appreciate the full impact of social choice, public choice and positive political economy for reconceptualising the basic building blocks of political liberalism". For Backhouse (2010, p. 137-8). "In such an environment, as Samuelson has admitted, it could be invaluable to present one's work as scientific, and the more technical, the better" As Bernstein puts it, "Anti-communism was a fundamental part of the process that defined what was (or was not) scientific" (Benrstein, 2001, p. 247). The more ideological the use of economics became, the more the need to present it as "scientific" and "objective". No other tool could serve this function better than mathematics, not least because of its prestige as a scientific instrument derived from the physical sciences.

# 8. Epilogue

In a review of Samuelson's *Foundation*, Kenneth Boulding (1948, p. 187) asks the following question:

Is economics an essentially mathematical science? ... The conflict between the mathematical and so-called 'literary' economists rages in our schools and can only be resolved, apparently, either by victory for one side or the other or by some agreed division of labor.

We know by now which side won a decisive victory in this battle. What is of interest in this context is Boulding's own answer, remembering that he writes in 1948. Although he points out that the battle is not (yet) over, he continues: "At this date mathematical economics is too ubiquitous to allow any question as to whether economics is a mathematical science" (p. 187). The formalist revolution, although still not fully complete, was already well under way. The programmatic aims of Jevons and Walras of some eighty years ago (the treatment of economics as a quantitative physico-mathematical science) have finally been brought to fruition and the reasons given by Boulding for this outcome are exactly the same as those given by Jevons eighty years ago: "Mathematics is a technique for the exposition and discovery of relationships among quantities. Economics clearly deals with quantitative concepts – prices, wages, outputs, incomes. In so far as it deals with quantity, it must be a mathematical science" (p. 187). And yet for the previous eighty years, not to mention the century before that, economics was essentially a *non*-mathematical science. Why this was the case is a question never posed by mainstream economists, the default answer being that economics had not yet developed into a "proper" science something that was bound to happen at some point or other. Our account, however, proves otherwise. The formalist revolution was not the inevitable outcome of the scientific maturation of our field of inquiry, but the result of a whole host of general and conjectural factors including social and institutional changes and ideology. After all, for the most part the history of our subject was not written in equations or even diagrams but verbally. Be that as it may, the persistence and extreme dominance of this phenomenon, peculiar to economics among the social sciences, does need further explanation a task that is beyond the scope of the present paper.

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